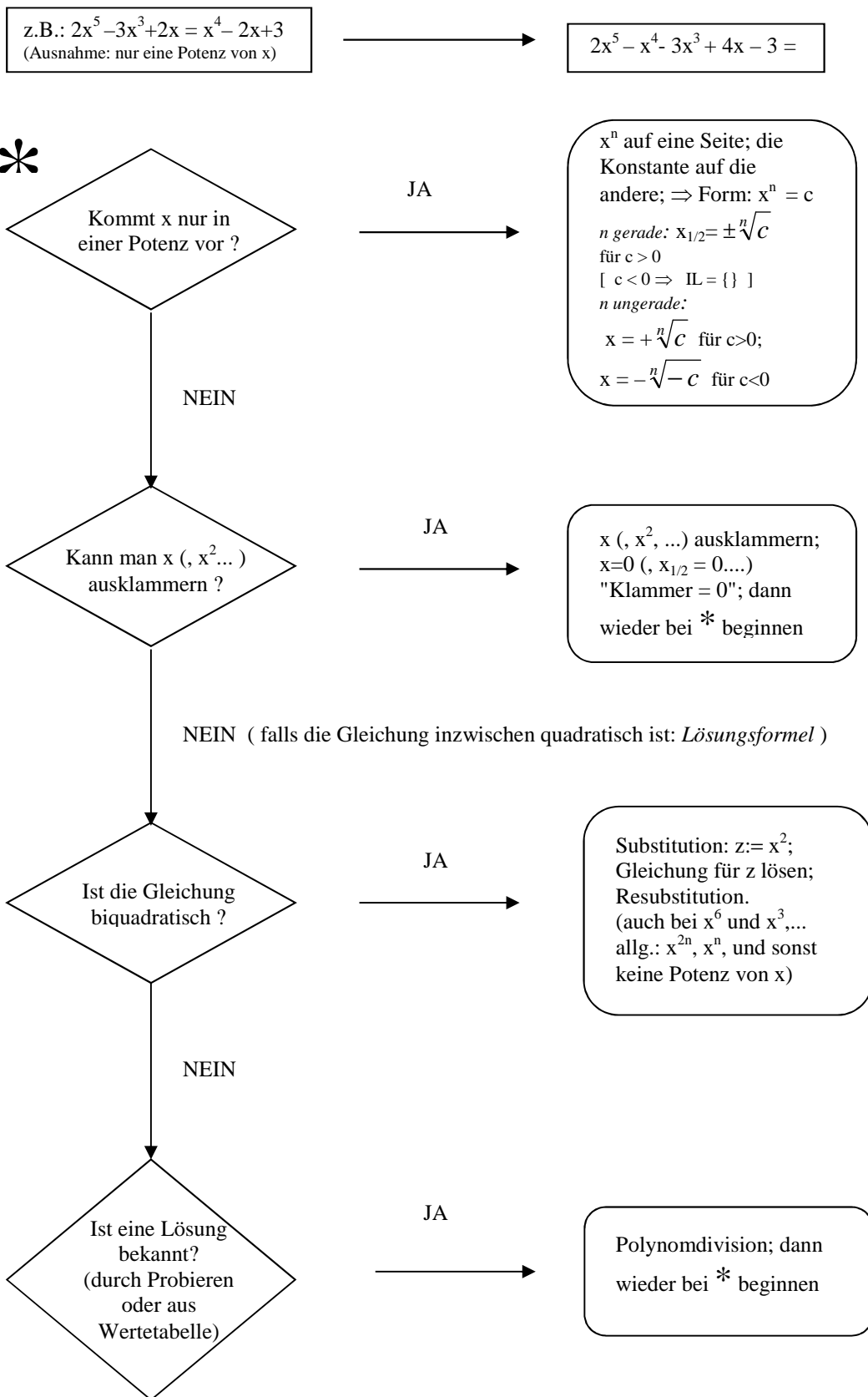


Rezept für das Lösen von Gleichungen höheren Grades

(Zunächst ist die Gleichung nach den Potenzen zu ordnen !)



Beispiele zum Lösen von Gleichungen höheren Grades

(Übungsaufgaben zur Polynomdivision und Aufgaben aus alten Prüfungen → nächste Seite)

B1.: $x^3 - 8 = 0$; $x^3 = 8$; $x = \sqrt[3]{8}$; $x = 2$

B2.: $x^4 - 625 = 0$; $x^4 = 625$; $x_{1/2} = \pm \sqrt[4]{625}$; $x_1 = 5; x_2 = -5$;

B3.: $x^5 + 32 = 0$; $x^5 = -32$; $x = -\sqrt[5]{32}$; $x = -2$

B4.: $x^4 + 625 = 0$; $x^4 = -625$; $x = \sqrt[4]{-625}$; $\mathbb{L} = \{\}$

B5.: $x^4 - 3x^3 + 2x^2 = 0$; $x^2(x^2 - 3x + 2) = 0$; $x_{1/2} = 0$
 mit Lösungsformel $x_3 = 2; x_4 = 1$

B6.: $x^4 - 4x^2 = 0$; $x^2(x^2 - 4) = 0$; $x_{1/2} = 0$;
 $x^2 - 4 = 0$; $x^2 = 4$; $x_{3/4} = \pm 2$

Ausklammern ist
weniger aufwändig als
die Substitution !!!

B7.: $x^4 + 4x^2 = 0$; $x^2(x^2 + 4) = 0$; $x_{1/2} = 0$;
 $x^2 + 4 = 0$; $x^2 = -4$; keine weiteren Lösungen



B8.: $x^4 - 5x^2 + 4 = 0$;
 $z := x^2$; $z^2 - 5z + 4 = 0$; mit Lösungsformel \Rightarrow
 $z_1 = 4$: $x^2 = 4$; $x_{1/2} = \pm 2$
 $z_2 = 1$: $x^2 = 1$; $x_{3/4} = \pm 1$

B9.: $x^4 - 3x^2 - 4 = 0$;
 $z := x^2$; $z^2 - 3z - 4 = 0$; mit Lösungsformel \Rightarrow
 $z_1 = 4$: $x^2 = 4$; $x_{1/2} = \pm 2$
 $z_2 = -1$: $x^2 = -1$; keine weiteren Lösungen

Methodenwechsel:

B10) $x^5 + 3x^4 - 3x^3 - 9x^2 - 4x - 12 = 0$
 $x_1 = -3$
 $(x^5 + 3x^4 - 3x^3 - 9x^2 - 4x - 12) : (x + 3) = x^4 - 3x^2 - 4$
 Nun geht's weiter wie beim Beispiel 9 !

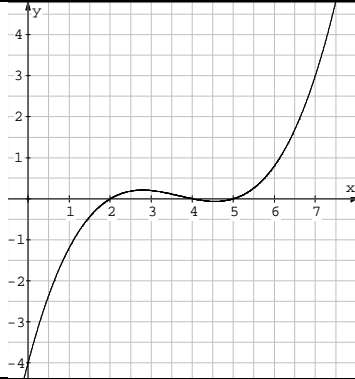
B11) $x^4 - 8x = 0$; $x \cdot (x^3 - 8) = 0$ $x_1 = 0$
 $x^3 - 8 = 0$;
 Nun geht's weiter wie beim Beispiel 1 !

| | nur Polynomdivision | Gemischt – alte Prüfungsaufgaben | |
|-----------|---|---|--------------------------|
| Nr | Funktion f mit f(x) = | Funktion f mit f(x) = | |
| 1 | $0,1x^3 - 1,1x^2 + 3,8x - 4$ | $\frac{1}{8}(x-4)^2(x+2)$ | Lösungen siehe Rückseite |
| 2 | $-x^3 + 6x^2 - 11x + 6$ | $-x^3 + 9x^2 - 17x - 3$ | |
| 3 | $0,25x^3 - 0,25x^2 - 2,5x - 2$ | $-\frac{1}{8} \cdot (x-4) \cdot (x+2)^2$ | |
| 4 | $-0,2x^3 + x^2 + 5,8x - 21$ | $\frac{1}{32}x^4 - x^2 + 8$ | |
| 5 | $-\frac{1}{2}x^3 - \frac{5}{2}x^2 - x + 4$ | $-\frac{20}{9}t^3 + 10t^2$ | |
| 6 | $x^3 - 7x - 6$ | $\frac{1}{8} \cdot (x^3 - 12x^2 + 32x)$ | |
| 7 | $0,5x^3 - 0,5x^2 - 2x + 2$ | $\frac{1}{27}(x+3)^2(x^2-9)$ | |
| 8 | $-\frac{1}{7}x^3 + x - \frac{6}{7}$ | $\frac{1}{9}x^3 - \frac{1}{3}x^2 - x + 3$ | |
| 9 | $-x^3 + 6x^2 + x - 30$ | $\frac{1}{8}x^4 + x^3 + \frac{9}{4}x^2$ | |
| 10 | $\frac{1}{3}x^3 - 3x^2 + \frac{26}{3}x - 8$ | $\frac{1}{24}x^4 - \frac{4}{9}x^3 + \frac{4}{3}x^2$ | |
| 11 | $-\frac{1}{6}x^3 - x^2 + \frac{1}{6}x + 5$ | $\frac{1}{27}(x^3 + 27x)$ | |
| 12 | $\frac{1}{6}(x^3 - 9x^2 + 24x - 20)$ | $-\frac{1}{12}x^3 - \frac{1}{2}x^2 + \frac{5}{4}x + \frac{25}{3}$ | |
| 13 | $-\frac{1}{9}(x^3 + x^2 - 9x - 9)$ | $\frac{4}{27}(x^4 + 4x^3)$ | |
| 14 | $0,2x^4 - 2x^3 + 7x^2 - 10x + 4,8$ | $\frac{1}{6}x^4 - x^2 - \frac{9}{2}$ | |
| 15 | $-\frac{1}{6}(x^4 - x^3 - 7x^2 + x + 6)$ | $\frac{1}{4}x^3 - 3x + 4$ | |
| 16 | $\frac{1}{3}x^4 + x^3 - x^2 - \frac{11}{3}x - 2$ | $-\frac{1}{8}(u^4 + 2u^3 - 8u^2)$ | |
| 17 | $-0,25x^4 + 0,25x^3 + 3x^2 + x - 4$ | $-\frac{1}{4}x^4 + x^3$ | |
| 18 | $\frac{1}{2}x^4 - x^3 - \frac{3}{2}x^2 + 4x - 2$ | $\frac{1}{4}(u^4 - 8u^3 + 12u^2)$ | |
| 19 | $-0,2(x^4 - 5x^3 - 7x^2 + 29x + 30)$ | $\frac{3}{16}x^3 - \frac{9}{4}x$ | |
| 20 | $0,25x^4 - 2,25x^2 + x + 3$ | $\frac{3}{16}x^3 - \frac{9}{4}x + 3$ | |
| 21 | $-\frac{1}{9}(x^4 - 4x^3 - 3x^2 + 10x + 8)$ | $\frac{1}{4}x^3 - 3x^2 + 9x$ | |
| 22 | $-\frac{1}{4}x^4 + \frac{5}{4}x^3 - \frac{3}{2}x^2 - x + 2$ | $-\frac{1}{4}(x^4 - 8x^2 - 9)$ | |
| 23 | $0,1(x^4 + x^3 - 7x^2 - 13x - 6)$ | $-\frac{1}{36}x^4 + x^2$ | |

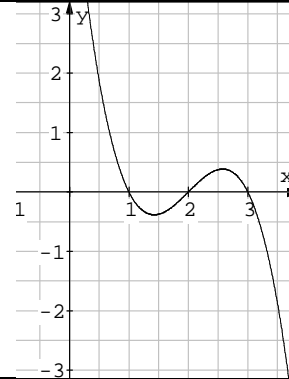
| | Lösungen Polynomdivision | Lösungen: alte Prüfungsaufgaben | |
|-----------|-------------------------------------|--|-----------------|
| Nr | Nullstellen | Nullstellen | Jahrgang |
| 1 | 2; 4; 5 | 4; 4; -2 | BOS 1998 |
| 2 | 1; 2; 3 | 3; $3 \pm \sqrt{10}$ | FOS 2000 |
| 3 | -2; -1; 4 | 4; -2; -2 | FOS 2002 |
| 4 | -5; 3; 7 | -4; -4; 4; 4 | FOS 2003 |
| 5 | -4; -2; 1 | 0; 0; 4,5 | FOS 2003 |
| 6 | -2; -1; 3 | 0; 8; 4 | FOS 2003 |
| 7 | -2; 1; 2 | -3; -3; -3; 3 | FOS 2004 |
| 8 | -3; 1; 2 | -3; 3; 3 | FOS 2004 |
| 9 | -2; 3; 5 | 0; 0 | NT FOS 1999 |
| 10 | 2; 3; 4 | 0; 0 | NT FOS 2000 |
| 11 | -5; -3; 2 | 0 | NT FOS 2001 |
| 12 | 2; 2; 5 | -5; -5; 4 | NT FOS 2003 |
| 13 | -3; -1; 3 | 0; 0; 0; -4 | NT FOS 2004 |
| 14 | 1; 2; 3; 4 | -3; 3 | NT FOS 1997 |
| 15 | -2; -1; 1; 3 | -4; 2; 2 | FOS 1992 |
| 16 | -3; -1; -1; 2 | 0; 0; 2; -4 | FOS 1992 |
| 17 | -2; -2; 1; 4 | 0; 0; 0; 4 | FOS 1993 |
| 18 | -2; 1; 1; 2 | 0; 0; 6; 2 | FOS 1993 |
| 19 | -2; -1; 3; 5 | 0; $\pm \sqrt{12}$ | FOS 1993 |
| 20 | -3; -1; 2; 2 | 2; 2; -4 | FOS 1993 |
| 21 | -1; -1; 2; 4 | 0; 6; 6 | FOS 1994 |
| 22 | -1; 2; 2; 2 | -3; 3 | FOS 1989 |
| 23 | -2; -1; -1; 3 | 0; 0; 6; -6 | FOS 1990 |

Graphen des Arbeitsblattes Polynomdivision (linke Seite) –Seite 1

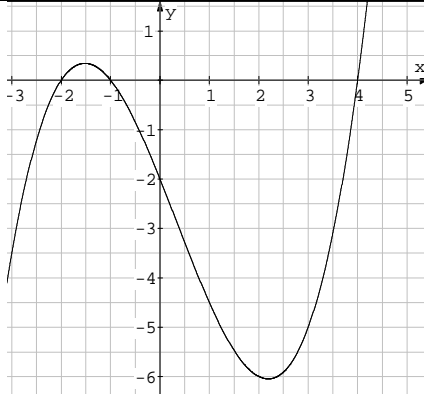
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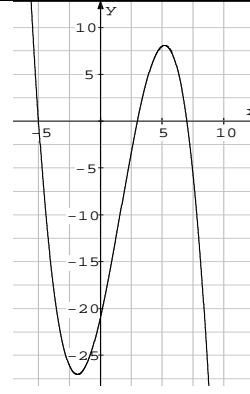
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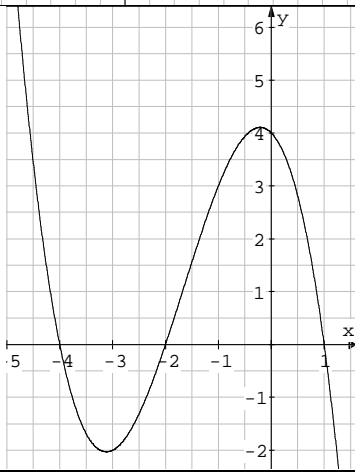
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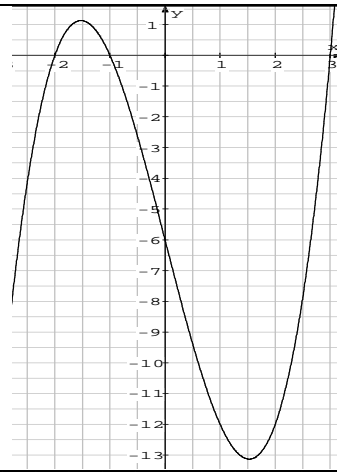
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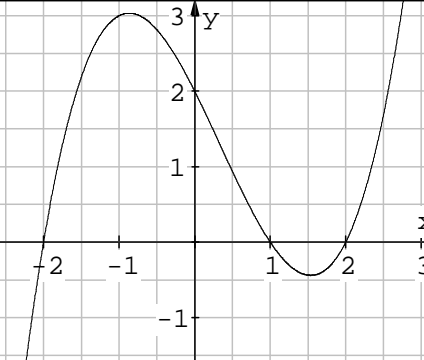
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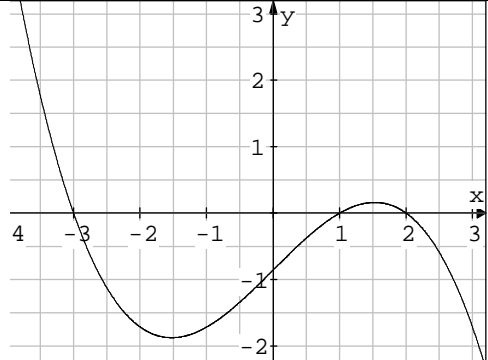
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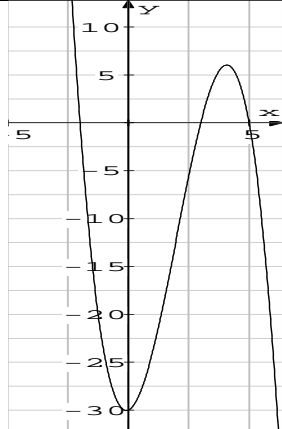


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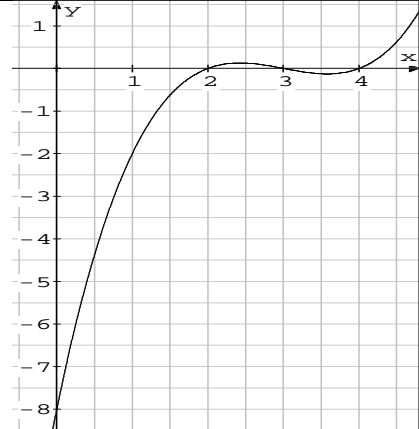


Graphen des Arbeitsblattes Polynomdivision (linke Seite) –Seite 2

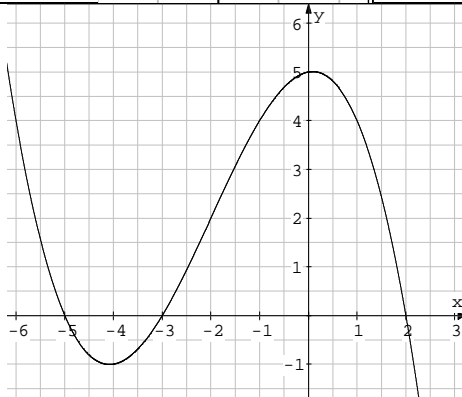
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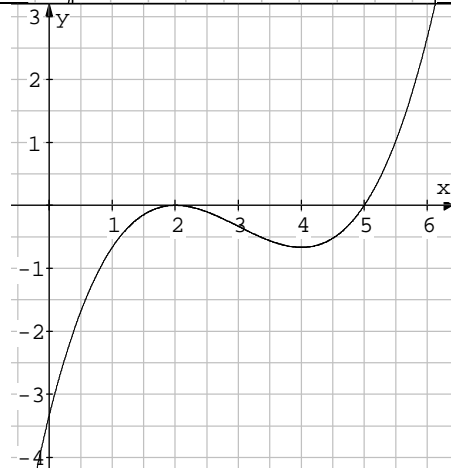
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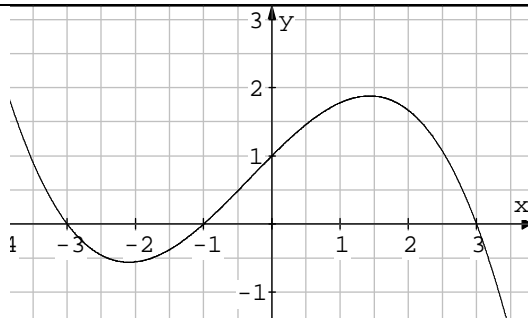
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12



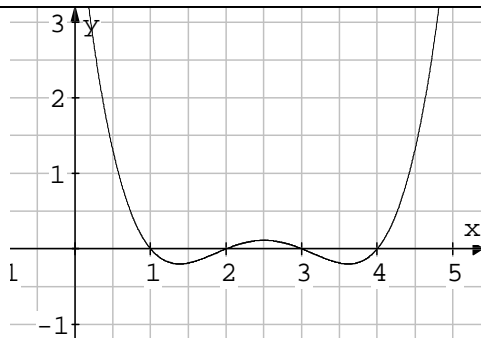
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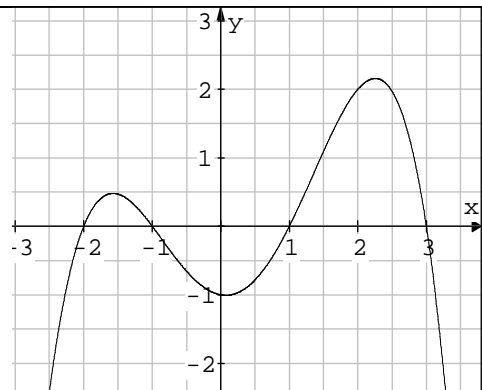
Ab jetzt:

Der Grad ist 4

14

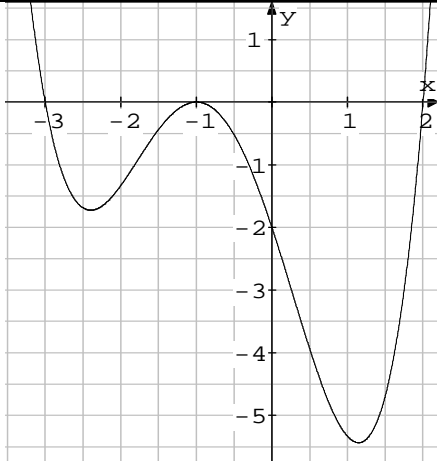


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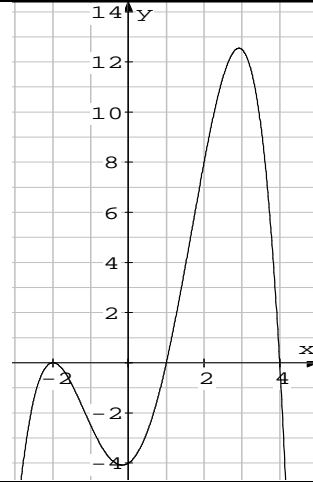


Graphen des Arbeitsblattes Polynomdivision (linke Seite) –Seite 3

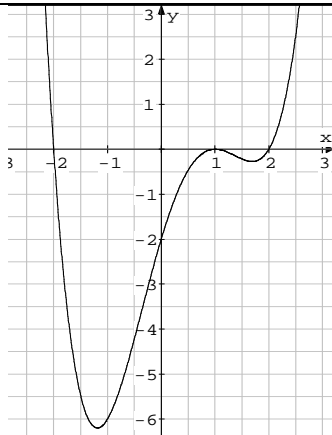
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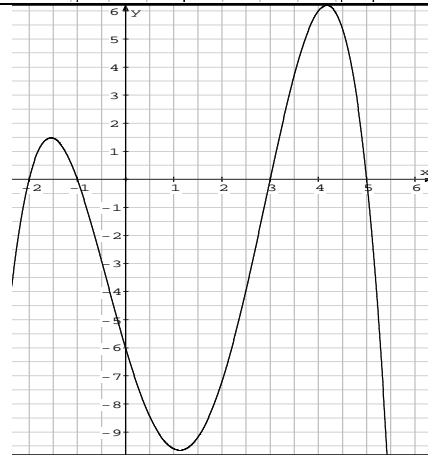
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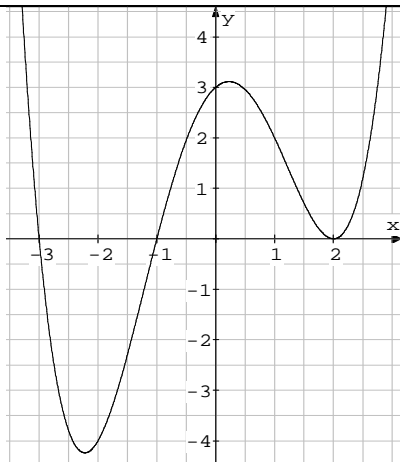
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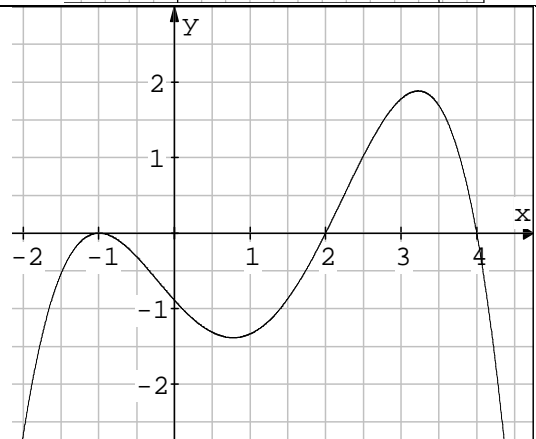
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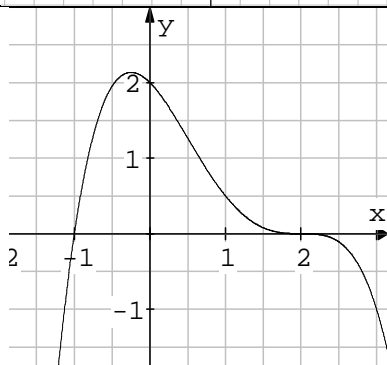
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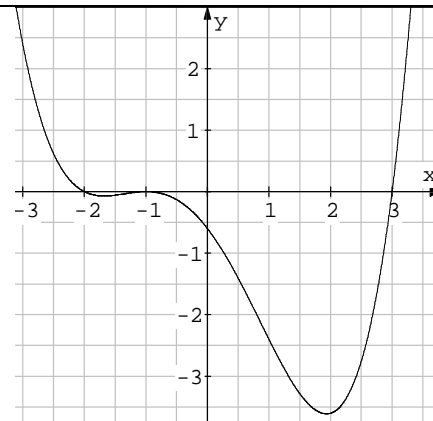
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22

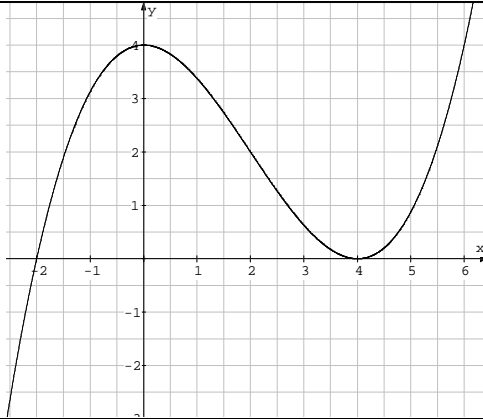


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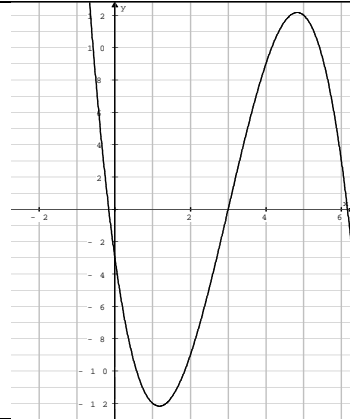


Graphen des Arbeitsblattes höherer Grad (rechte Seite) – Prüfungsaufgaben – Seite 1

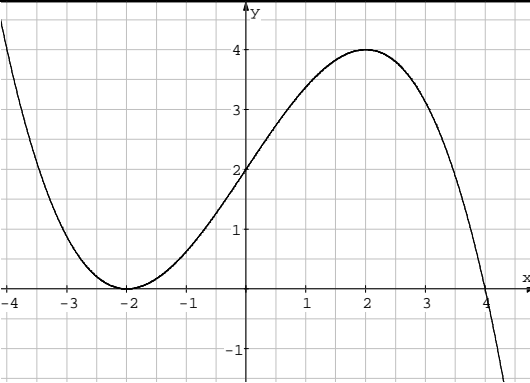
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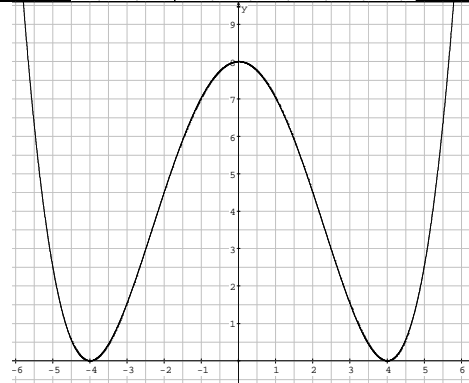
2



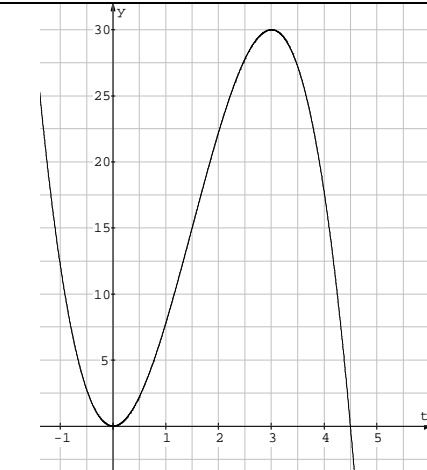
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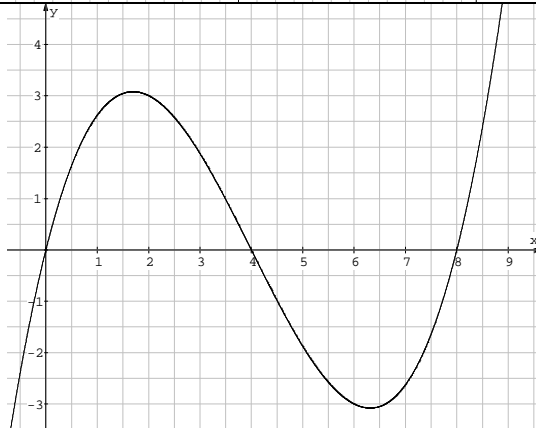
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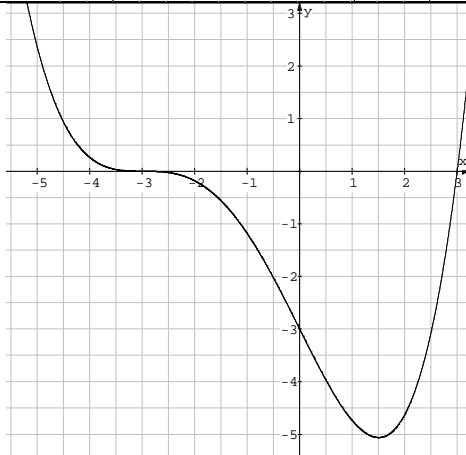
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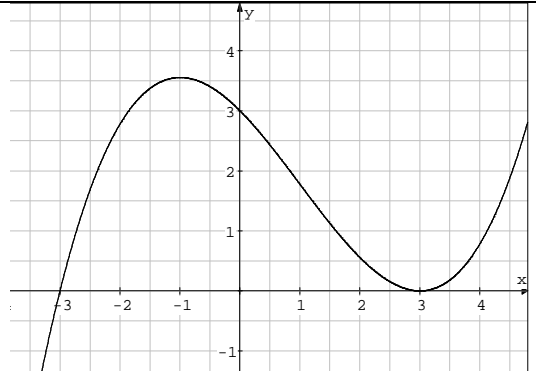
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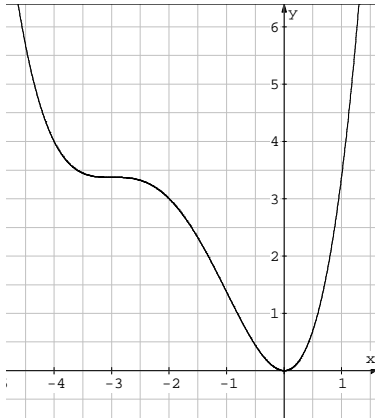
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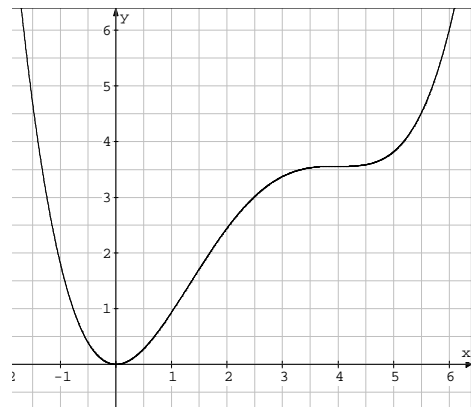
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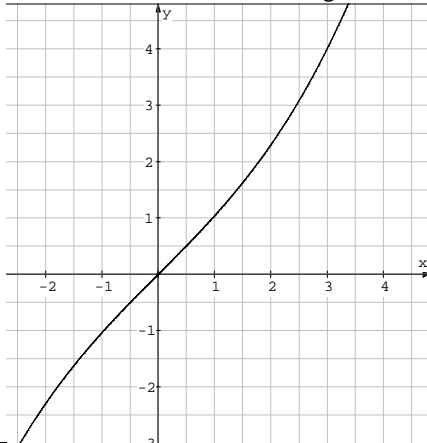
9 Differentialrechnung erforderlich!



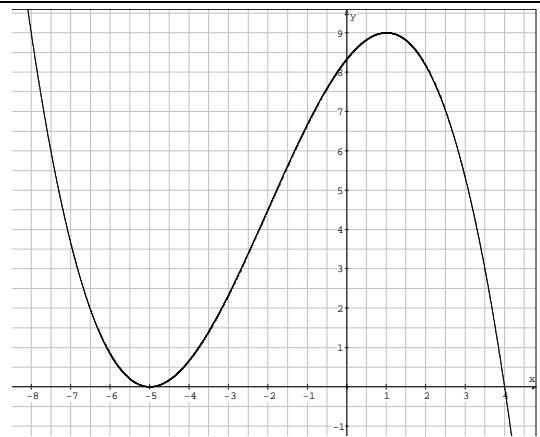
10 Differentialrechnung erforderlich!



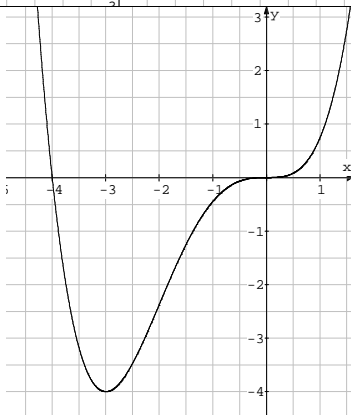
11 Differentialrechnung erforderlich!



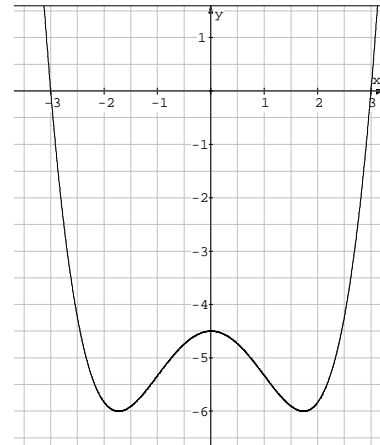
12



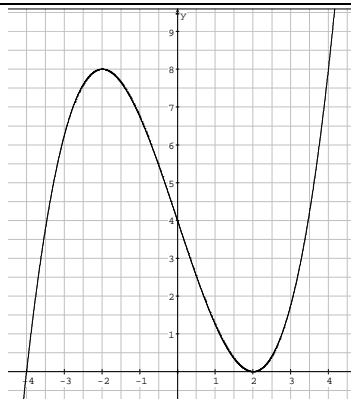
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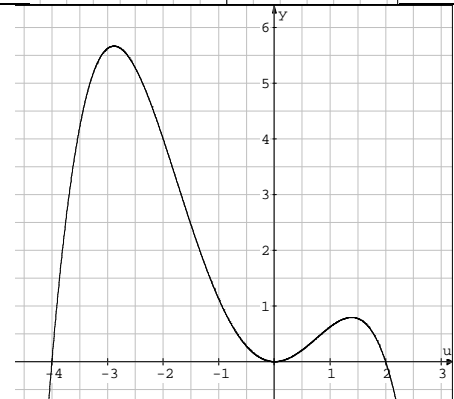
14 Differentialrechnung erforderlich!



15

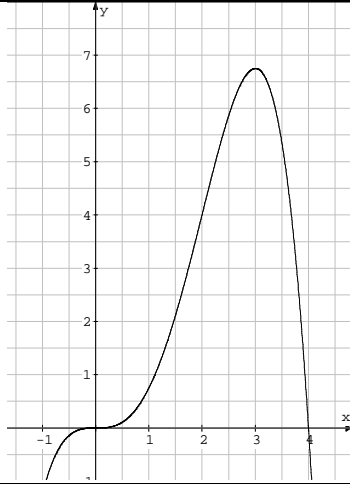


16

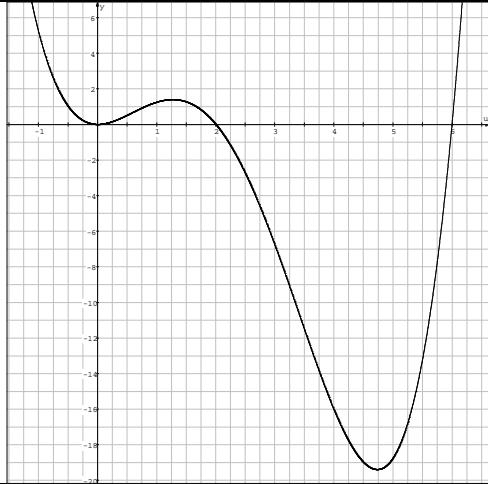


Graphen des Arbeitsblattes höherer Grad – Prüfungsaufgaben – Seite 3

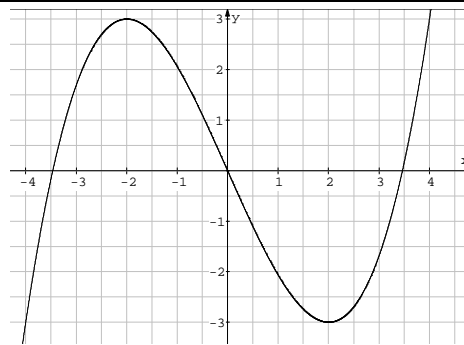
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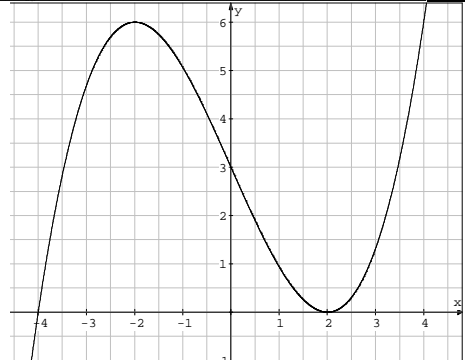
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19

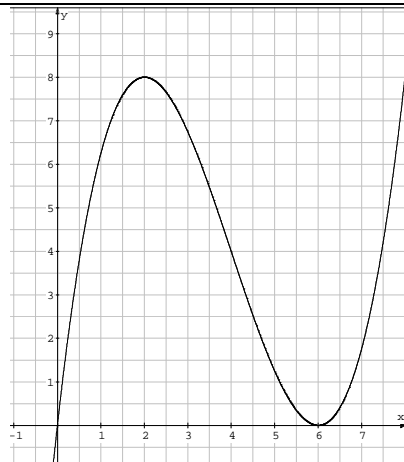


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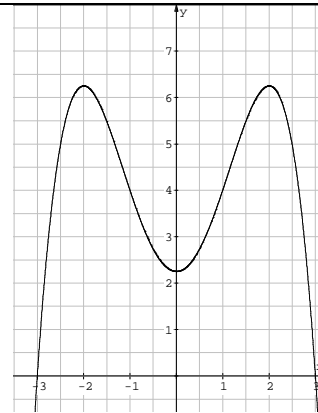


Die beiden Graphen (19, 20) sind nicht ohne Grund sehr ähnlich!!!

21



22



Differentialrechnung erforderlich!

23

